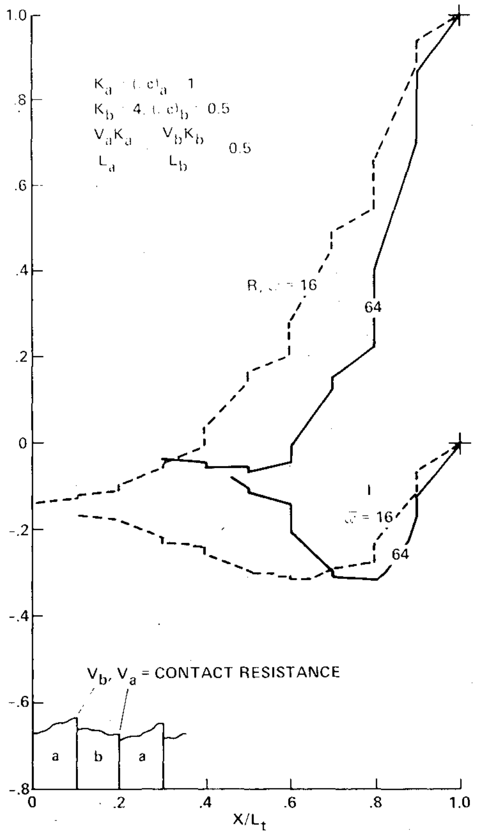

Fig. 3 Temperature profiles in a ten-layer composite.

diagram for a two-material, ten-layer composite with interface contact resistances. Specifications are self-explanatory in Fig. 3. Note the assumed dimensionless value of $(V_j k_j / L_j) = 0.5$, which makes the contact resistances unequal in their physical units, thus causing uneven gaps in the curves.

IV. Composite Spheres and Cylinders

For composite spheres consisting of a number of shells, the method of analysis is essentially unchanged because of similar eigenfunctions in individual sphere-layer solutions. All that is needed is to modify Eq. (2) by dividing the right-hand side by a local radius and interpreting x_j as the local radius minus the inner radius of the jth shell. The resulting recurrence relation remains largely intact. As for composite cylinders, however, the functional structure of Eq. (2) is changed into that of Bessel functions in complex arguments, with some numerical complications that can be dealt with.

Acknowledgment

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References

[1]Carslaw, H.S. and Jaeger, J.C., *Conduction of Heat in Solids*, Oxford University Press, New York, 1959.

[2]Mikhailov, M.D., Ozisik, M.N., and Vulchanov, N.L., "Diffusion in Composte Layers with Automatic Solution of the Eigenvalue Problem," *International Journal of Heat and Mass Transfer*, Vol. 26, Aug. 1983, pp. 1131–1141.

[3]Maewal, A., Bache, T.C., and Hegemier, G.A., "A Continuum Model for Diffusion in Laminated Composite Media," *Journal of Heat and Transfer*, Feb. 1976, pp. 133–138.

[4]Schimmel, W.P., Beck, J.V., and Donaldson, A.B., "Effective Thermal Diffusivity for a Multimaterial Composite Laminate," *Journal of Heat Transfer*, Aug. 1977, pp. 466–470.

Prediction of Film Boiling Wakes Behind Cylinders in Cross Flow

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Nomenclature

C_N = cavitation number, see Eq. (3)
D = cylinder diameter
Fr = Froude number, see Eq. (5)
g = gravity
p = pressure
Δp = pressure difference
R = cylinder radius
T = temperature
ΔT_B = temperature difference, $T_{sat} - T_B$
u = velocity in the x direction
V = cross-flow velocity
W = complex potential function, $\phi + i\psi$
WL = wake length
WT = wake thickness
x = cylindrical surface coordinate
y = cylindrical surface normal coordinate
Z = complex function, $x + iy$
θ = angle measured from the stagnation point
μ = viscosity
ρ = density
δ = vapor film thickness
ϕ = velocity potential function
ψ = stream function

Subscripts

B = bulk
l = liquid
obs = observed
s = separation point
sat = saturation
th = theoretical
v,vap = vapor
wk = wake
w = wall
∞ = freestream

Introduction

FILM boiling is often encountered in cases such as handling of cryogenic fluids, quenching of metal parts, cooling of rocket nozzles, etc. A film of vapor separates the liquid from its heat source during this boiling mode. Film boiling can be classified into two main groups—natural convection (or pool) and forced convection (or flow) film boiling. There is no forced relative motion between the liquid and the hot surface in pool boiling. In contrast, flow boiling involves relative motion between the liquid and the surface.

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Bromley and co-workers[1,2] were the first to investigate flow film boiling. They developed a model that consists of a body whose front stagnation point is covered by a thin laminar vapor film with a smooth liquid/vapor (l-v) interface. The liquid is assumed to undergo potential flow over the vapor film. The vapor forms a thick vaporous wake at the trailing edge of the body.

Epstein and Hauser[3] analyzed film boiling near the forward stagnation point of a horizontal cylinder, accounting for viscous effects at the l-v interface. They found that shear might affect the liquid motion without influencing the heat transfer in the saturated case. The effect of shear becomes negligibly small for large subcooling in the liquid.

Witte and Orozco[4] performed a similar analysis but neglected shear in the liquid. They allowed the vapor film to vary around the cylinder by imposing the liquid pressure on the vapor. This yields nonlinear velocity in the vapor; the use of such a profile yields improved heat-transfer predictions. Also, the point of vapor flow reversal in the vapor film, which apparently leads to the formation of a wake, can be predicted.

Even the theory of Ref. 4 does not fully predict experimental heat transfer during flow film boiling, because the heat transfer in the wake is totally neglected. Understanding heat transfer in the wake might be easier if we could predict the extent (length and thickness) during flow boiling.

This Note describes the model used to predict film boiling wakes behind cylinders in cross flow and compares the results to limited Freon-11 data.

Theory

Wakes formed behind submerged bodies due to film boiling over the front of the body resemble wakes due to cavitation. Indeed, "supercavitation" wakes seem quite similar to film boiling wakes. This similarity is exploited in the following development, which leads to a two-stage model. The first stage uses the boundary-layer analysis of Ref. 4 to define the flow over the front of a cylinder; the second stage uses a cavitation model to obtain the wake profile.

Boundary-Layer Analysis

In Ref. 4, the boundary-layer equations were coupled to the energy equation to give a differential equation for the film thickness. The model predicts the point of vapor flow separation from the cylindrical surface as well. For brevity, only a brief sketch of the theory is given herein.

It was assumed that a smooth l-v interface with laminar vapor and liquid flows exists, that the vapor film is thin compared to the radius of the cylinder, and that the liquid velocity is unaffected by vapor drag at the l-v interface. Additionally, the liquid and vapor were treated as incompressible and buoyant forces, inertial and convective effects in the vapor, and thermal radiation were neglected.

The vapor velocity is controlled by the pressure gradient in the liquid. The momentum balance gives the following equation for the vapor velocity:

$$u = 2\frac{\rho_1 V^2}{\mu_v R}\sin\theta\,\cos\theta(y\delta - y^2) + 2\left(\frac{y}{\delta}\right)V\sin\theta \qquad (1)$$

The temperature across the vapor film is assumed to be linear because of the thinness of the vapor film. The liquid energy equation, solved according to the method of Sideman,[5] gives the temperature distribution in the liquid. The mass-energy balance yields a differential equation that describes the vapor thickness around the cylinder.

Differentiation of Eq. (1) yields the angle at which the vapor flow reverses in the vapor film. This is the angle of separation θ_s—i.e., where the wake begins. The following equation gives θ_s in terms of system parameters:

$$\cos\theta_s = -\frac{\mu_v R}{\rho_1 V\delta_s^2} \qquad (2)$$

Typical values are shown in Fig. 1, which is a plot of vapor film thickness around a cylinder for various degrees of subcooling.

Cavitation Model—Potential Flow

A potential flow cavitation model was adapted to predict cavities behind heated cylinders. The cavitation number, defined as

$$C_N = \frac{p_\infty - p_{\rm wk}}{\frac{1}{2}\rho_1 V^2} \qquad (3)$$

characterizes cavitation wakes. The lower C_N is the greater the tendency for cavitation. Several models for prediction of cavities exist for known values of C_N and θ_s.

Figure 2 shows a "re-entrant jet" model as described in Refs. 6 and 7. The infinite flow streaming past the cylinder breaks up into a fluid jet that streams back toward the cylinder's rear stagnation point. This model works reasonably well to predict the dimensions of cavitation wakes and also the form drag of bodies undergoing cavitation.

The equations describing cavity flow are solved by using a conformal mapping of the actual flowfield. The velocity

$$W(Z) = \phi + i\psi \qquad (4)$$

where ϕ and ψ are the velocity potential and stream function, respectively, is mapped onto the ϵ plane; see Ref. 7.

The procedure is to find an appropriate mapping function in the ϵ domain and transform it back onto the actual flowfield. The details of this development are given in Ref. 8.

The result of the analysis was that a solution can be found if the following parameters are known: 1) the point of separation (known from Ref. 4); 2) the shape of the wetted obstacle

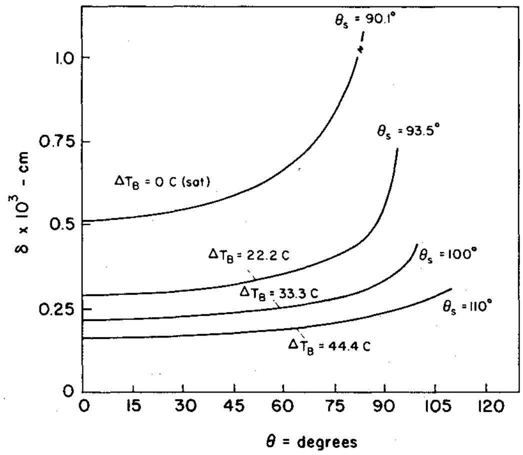

Fig. 1 Variation of the vapor film thickness over a 6.35 cm cylinder for the following conditions: Freon-113, $T_{\rm sat}$ = 50.6°C, V = 6.8 m/s, T_w = 140.6°C.

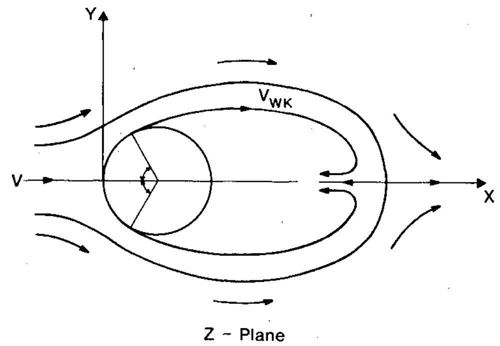

Fig. 2 Re-entrant jet cavitation model.

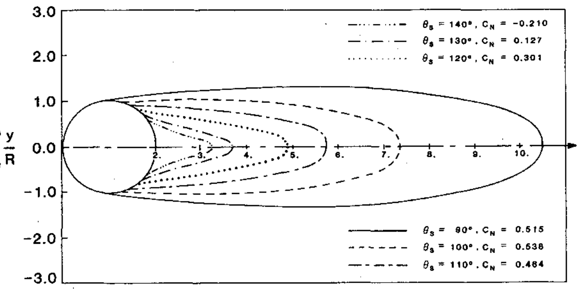

Fig. 3 Calculated wake shapes for various values of θ_s and C_N.

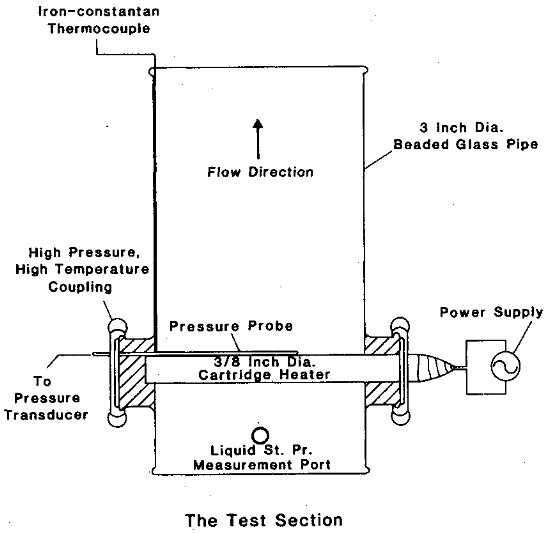

Fig. 4 Heater in place in the test section.

(the diameter of the cylinder); and 3) the cavitation number (known from wake pressure). Qualitative solutions can be determined by assuming certain compatible values of C_N and θ_s; Fig. 3 shows several wakes calculated for *assumed* values of C_N and θ_s. This plot shows that the calculated wakes agree well with qualitative observations of film boiling wakes. Stevens (see Ref. 3) observed that θ_s increases as the liquid subcooling increases. Figure 3 shows that the wake length (WL) and thickness (WT) are reduced by increasing θ_s. This agrees with the observations of Stevens.

The wake pressure p_{wk} is not known a priori for a given flow boiling situation. Thus, it was necessary to devise an experiment in which p_{wk} could be measured.

Experimental Apparatus

A closed flow loop was used for the experiments on flow film boiling across a cylindrical heater. The loop provides a flat, laminar velocity profile at the test section. Additional details of the flow loop can be found in Ref. 9.

The test heater was a 600 W cartridge heater with an outside diameter of 0.953 cm and length of 7.62 cm. The heater was mounted between two 1.91 cm diameter ports diametrically opposed in a 7.62-cm i.d. glass pipe that made up the test section as shown schematically in Fig. 4.

The surface temperature of the heater was measured by an iron-constantan thermocouple attached to its trailing side. Heater power was controlled by a Variac power supply feeding a step-up transformer.

The static pressure of the liquid was measured through a tap located on the test section wall opposite the leading edge of the cylinder. The Δp between the liquid and the wake was measured by a 0-34.47 kPa (0-5 psia) differential transducer connected to a sealed 1.25 mm stainless steel tube situated in the wake behind the cylinder. A small hole was drilled near the sealed end, perpendicular to the tube axis, to sense the pressure.

Further details of the apparatus are given in Ref. 8.

Table 1 Summary of experimental data

Heater wall temp., °C	Liquid bulk temp., °C	Liquid velocity, m/s	System pressure, MPa	Pressure difference $p_\infty - p_{wk}$, cm of H_2O	Heater power, W
119	26	1.95	0.138	17.5	620
99	27	2.13	0.143	17.0	620
99	28	1.95	0.147	19.5	620
101	28	1.68	0.148	21.7	620
104	29	1.19	0.161	26.5	620
107	29	0.94	0.159	28.0	620
296	31	2.16	0.161	23.0	600
189	32	2.16	0.161	24.0	560
193	31	2.01	0.157	25.0	520
216	31	1.28	0.153	29.0	420

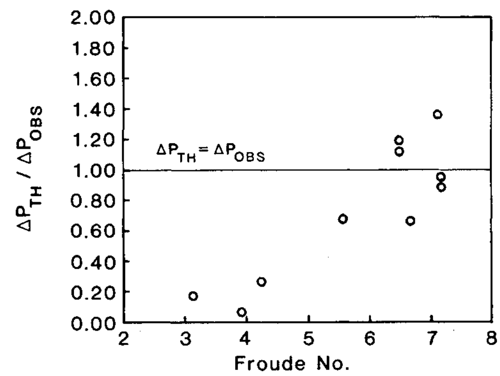

Fig. 5 Comparison of theoretical and observed wake pressures.

Experiments

By adjusting the heater power level and flow velocity, different points of stable film boiling in Freon-11 were achieved. At each point, T and p measurements were obtained and the wakes were photographed to determine WL. Table 1 summarizes the data that were obtained. The range of velocity was limited by the capability of the flow loop.

Results and Discussion

Figures 5 and 6 show that the ratio of theoretical values, such as p_{wk} and WL, to similar experimental values scatter about unity at higher liquid velocities, but fail to do so at lower velocities. (Unity would indicate perfect agreement between theory and observation.) According to Ref. 10, gravity influences the two-phase liquid/wake flow as long as the Froude number, defined as

$$Fr = \left[\frac{\rho_l V^2}{gD(\rho_l - \rho_v)}\right]^{1/2} \tag{5}$$

is less than 10 for upwardly directed flows. The Fr was in the range of about 3-7, so none of the data obtained in this apparatus are entirely free of gravity effects. The theory takes no account of gravity at all, so the data at low Fr should not compare too favorably with the theory.

Figure 5 shows that the theoretical Δp required for an observed WL is substantially less than the experimental ones at low Fr. The reason is that as $Fr \rightarrow 0$, the phenomenon ap-

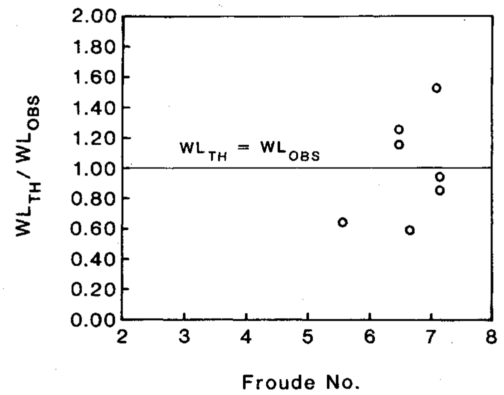

Fig. 6 Comparison of theoretical and observed wake lengths.

proaches pool film boiling, in which vapor is removed from the wake by a Taylor instability mode. The potential flow theory does not account for this behavior; thus, only data at $Fr > 5$ were retained in Fig. 6.

In our opinion, the scatter in the data is largely caused by the competing effects of buoyancy and inertia. Systematic uncertainty due to pressure sensor and recording errors was estimated at $\pm 5\%$, much less than the scatter inherent in the data. Limitations on liquid velocity would not allow data to be obtained in the $Fr \geq 10$ range.

The theory is not very accurate near the end of the wake because of the artificiality of the wake closure model. Taking this into account, the WL predictions shown in Fig. 6 exhibit reasonable conformity with experiment. However, the data are scattered; they cluster about the line of perfect agreement with a variance of 10%. Data for $Fr \geq 10$ are needed for more conclusive evidence of the applicability of the model, but could not be obtained with our apparatus.

WT at the point of separation could not be measured because the heater mounts obscured the end view of the heater. However, an "effective" WT_{obs} was calculated based on the Δp required to predict WL_{obs}. A comparison of this value to WT_{th} calculated from the measured Δp showed that WT is less sensitive to p_{wk} than is WL and is predicted by the theory quite well. This is so because the theory is more accurate near the trailing edge of the cylinder.

Conclusions

The conclusions that have been reached in this study are:

1) Film boiling wakes are not substantially different than cavitation wakes. However, for prediction, the proper θ_s and p_{wk} must be known.

2 The re-entrant jet theory predicts film boiling wake behavior reasonably well.

3) Wake shapes computed from the theory agree qualitatively with observations of subcooled flow film boiling wakes. The theory predicts that WL and WT are reduced as subcooling in the liquid is increased at a given liquid velocity, which agrees with prior observations.

Acknowledgments

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References

[1]Bromley, L.A., "Heat Transfer in Stable Film Boiling," *Chemical Engineering Progress,* Vol. 46, 1950, pp. 221-227.

[2]Bromley, L.A., LeRoy, N.R., and Robbers, J.A., "Heat Transfer in Forced Convection Film Boiling," *Industrial and Engineering Chemistry,* Vol. 45, 1953, pp. 2639-2645.

[3]Epstein, M. and Hauser, G., "Subcooled Forced Convection Film Boiling in the Forward Stagnation Region of a Sphere or Cylinder," *International Journal of Heat and Mass Transfer,* Vol. 23, 1980, pp. 179-189.

[4]Witte, L.C. and Orozco, J., "The Effect of Vapor Velocity Profile Shape on Flow Film Boiling from Submerged Bodies," *Journal of Heat Transfer,* Vol. 106, No. 1, Feb. 1984, pp. 191-197.

[5]Sideman, S., "The Equivalence of the Penetration Theory and Potential Flow Theories," *Industrial and Engineering Chemistry,* Vol. 58, No. 2, 1966, pp. 54-68.

[6]Gurevich, M.I., *Theory of Jets in Ideal Fluids* (translated from Russian), Academic Press, New York, 1965, pp. 181-242.

[7]Chaplygin, S.A., "Gas Jets," *Scientific Memoirs,* Moscow University, Moscow, 1962 (translated as NACA TM 1063, 1944).

[8]Kaul, R., "Hydrodynamics of Wakes Caused by Film Boiling," MS Thesis, Dept. of Mechanical Engineering, University, of Houston, TX, May 1985 (also AIAA Paper 85-1043, June 1985).

[9]Orozco, J., "Flow Film Boiling from Submerged Bodies," Ph.D. Dissertation, Dept. of Mechanical Engineering, University of Houston, TX, Dec. 1984.

[10]Lienhard, J.H., Eichhorn, R., Hasan, M.M., and Hasan, M.Z., "Boiling Burnout During Crossflow over Cylinders, Beyond the Influence of Gravity, *Journal of Heat Transfer,* Vol. 103, 1981, pp. 478-484.

Determination of the Cross-Sectional Temperature Distribution and Boiling Limitation of a Heat Pipe

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PRESENTLY, the primary means for rejecting heat from orbiting spacecraft is through a space radiator system composed of a series of single-phase fluid loops. Because this system uses a mechanically pumped coolant circuit, it is one whose long-mission reliability is low due to failure resulting from penetration by a single meteroid. The reliability of the system can be increased, but this results in a large increase in total weight. Hence, there is need for the development of a long-life heat rejection system suitable for long-term, high-power missions.

One solution to this problem is to develop a large modular radiator system that can be assembled during orbit from a number of standard components. This "space-constructible" radiator system would fulfill the needs and demands of large long-lived heat rejection systems by allowing the radiators to be built up for any desired heat load capacity. The planned system consists of individual heat pipes with radiator fins attached. These heat pipes would form the radiator and would be "plugged in" to contact heat exchangers designed to carry heat from the habitation module to the radiator through a centralized fluid loop.

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